Matrices Equations

Addition and Subtraction

- ✓ Only matrices with the same dimensions can be added/subtracted.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \pm \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a \pm e & b \pm f \\ c \pm g & d \pm h \end{bmatrix}$$

Multiplication

- ✓ To multiply by a scalar, use the distributive property.

$$a \begin{bmatrix} b & c \\ d & e \end{bmatrix} = \begin{bmatrix} ab & ac \\ ad & ae \end{bmatrix}$$

- ✓ Matrices can only be multiplied if the number of columns of the first matrix is the same as the number of rows of the second matrix.
- ✓ The number of rows in the product matrix is the same as the number of rows in the first matrix. The number of columns in the product is the same as the number of columns in the second.
- ✓ To create an element in the product matrix, move across a row and down a column.

$$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \times \begin{bmatrix} g \\ h \\ i \end{bmatrix} = \begin{bmatrix} ag + bh + ci \\ dg + eh + fi \end{bmatrix}$$

$$[\mathbf{2} \times 3] \times [3 \times \mathbf{1}] = [\mathbf{2} \times \mathbf{1}]$$

Determinants

- ✓ "Square" matrices have the same numbers of rows and columns.
- ✓ Only square matrices have determinants.
- ✓ To find the determinant of a 2×2 matrix, find the product of the elements down to the right, then subtract the product of the elements up to the right.

$$\text{If } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \text{ then } \mathbf{det}\boldsymbol{A} = \begin{vmatrix} \boldsymbol{a} & \boldsymbol{b} \\ \boldsymbol{c} & \boldsymbol{d} \end{vmatrix} = \boldsymbol{ad} - \boldsymbol{cb}$$

- ✓ To find the determinant of a 3×3 matrix (easily), begin by rewriting the first two columns to the right of the matrix. Find the sum of the products down to the right, then subtract the sum of the products up to the right.

$$\text{If } A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}, \text{ then } \left|\begin{matrix} \boldsymbol{a} & \boldsymbol{b} & \boldsymbol{c} \\ \boldsymbol{d} & \boldsymbol{e} & \boldsymbol{f} \\ \boldsymbol{g} & \boldsymbol{h} & \boldsymbol{i} \end{matrix}\right| \begin{matrix} \boldsymbol{a} & \boldsymbol{b} \\ \boldsymbol{d} & \boldsymbol{e} \\ \boldsymbol{g} & \boldsymbol{h} \end{matrix} = (\boldsymbol{aei} + \boldsymbol{bfg} + \boldsymbol{cdh}) - (\boldsymbol{gec} + \boldsymbol{hfa} + \boldsymbol{idb})$$

Cramer's Rule

- ✓ Cramer's Rule can be used to solve systems of equations.
- ✓ For systems of equations in two variables, create three matrices: one general, one for the first variable, and one for the second.
- ✓ The general matrix is made from the coefficients of the variables.
- ✓ The x matrix is made by replacing the x column with the constants.

- ✓ The y matrix is made by replacing the y column with the constants.

$$Ax + By = C$$
$$Dx + Ey = F$$

$$M = \begin{bmatrix} A & B \\ D & E \end{bmatrix} \quad M_x = \begin{bmatrix} \boldsymbol{C} & B \\ \boldsymbol{F} & E \end{bmatrix} \quad M_y = \begin{bmatrix} \boldsymbol{A} & C \\ \boldsymbol{D} & F \end{bmatrix}$$

$$x = \frac{\det M_x}{\det M} \quad y = \frac{\det M_y}{\det M}$$

- ✓ For systems of equations in three variables, create four matrices: one general and one for each variable.
- ✓ The general matrix is made from the coefficients of the variables.
- ✓ The x matrix is made by replacing the x column with the constants.
- ✓ The y matrix is made by replacing the y column with the constants.
- ✓ The z matrix is made by replacing the z column with the constants.

$$Ax + By + Cz = D$$
$$Ex + Fy + Gz = H$$
$$Ix + Jy + Kz = L$$

$$M = \begin{bmatrix} A & B & C \\ E & F & G \\ I & J & K \end{bmatrix} \quad M_x = \begin{bmatrix} \boldsymbol{D} & B & C \\ \boldsymbol{H} & F & G \\ \boldsymbol{L} & J & K \end{bmatrix} \quad M_y = \begin{bmatrix} A & \boldsymbol{D} & C \\ E & \boldsymbol{H} & G \\ I & \boldsymbol{L} & K \end{bmatrix} \quad M_z = \begin{bmatrix} A & B & \boldsymbol{D} \\ E & F & \boldsymbol{H} \\ I & J & \boldsymbol{L} \end{bmatrix}$$

$$x = \frac{\det M_x}{\det M} \quad y = \frac{\det M_y}{\det M} \quad z = \frac{\det M_z}{\det M}$$

Inverse Matrices

- ✓ Only square matrices have inverses.
- ✓ Inverse matrices can be used to solve equations in which one factor matrix and the product matrix are known but the second factor matrix is not — in algebraic terms, when $AX = B$, and A and B are known matrices.
- ✓ The product of a matrix A and its inverse, denoted by A^{-1}, is an identity matrix, denoted by I. The identity matrix is one which, when multiplied by any other matrix (by which it *can* be multiplied), yields a matrix with the same elements as the other matrix. It is analogous to multiplying a number by 1.
- ✓ The identity matrices for 2×2 and 3×3 matrices are as follows:

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ and } I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

If A is a square matrix, then $AI = A$ and $IA = A$.

- ✓ The inverse of a 2×2 matrix can be found using the following formula:

$$\text{If } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \text{ then } A^{-1} = \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

- ✓ The inverse of a 3×3 matrix can be found using the following formula:

$$\text{If } A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}, \text{ then } A^{-1} = \frac{1}{\det A} \begin{bmatrix} ei - fh & ch - bi & bf - ce \\ fg - di & ai - cg & cd - af \\ dh - eg & bg - ah & ae - bd \end{bmatrix}$$

- ✓ To solve an equation involving two known matrices and one unknown . . .

$$AX = B$$

Find A^{-1}. Multiply both sides of the equation by that matrix. A^{-1} must come first on the right side of the equation.

$$(\boldsymbol{A^{-1}})AX = (\boldsymbol{A^{-1}})B$$

The product of A and A^{-1} is the identity matrix, which simplifies the left side of the equation to X.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} X = A^{-1}B$$

$$\boldsymbol{X = A^{-1}B}$$

- ✓ Inverse matrices can be used to solve equations in more than one variable.

$$Ax + By = C$$
$$Dx + Ey = F$$

Create a matrix made up of the coefficients of the variables and another of the constants.

$$\begin{bmatrix} A & B \\ D & E \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} C \\ F \end{bmatrix}$$

Multiply both sides of the equation by the inverse of the coefficient matrix. The result is that

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{\begin{vmatrix} A & B \\ D & E \end{vmatrix}} \begin{bmatrix} E & -B \\ -D & A \end{bmatrix} \begin{bmatrix} C \\ F \end{bmatrix}$$

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